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Deformed Bialgebra of Diffeomorphisms

Marija Dimitrijević

Max-Planck-Institut für Physik, München, Germany
University of Belgrade, Faculty of Physics,
Belgrade, Serbia and Montenegro

with P. Aschieri, C. Blohmann, F. Meyer, P. Schupp and J.Wess

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Outline

- Definition of deformed (noncommutative) spaces
- Derivatives on deformed spaces
- Representation on space of commutative variables ($\star$-product representation)
- Bialgebra of diffeomorphisms; deformed Lorentz transformations
- Summary and outlook

Deformed (noncommutative) space

Commutative space generated by coordinates x^μ, $\mu = 1, \dots n$ with

$$[x^\mu, x^\nu] = 0.$$

Noncommutative space $\hat{\mathcal{A}}_{\hat{x}}$, generated by $\hat{x}^\mu$ coordinates, $\mu = 1, \dots n$ such that:

$$[\hat{x}^\mu, \hat{x}^\nu] = \Theta^{\mu\nu}(\hat{x}). \tag{1}$$

It is the associative free algebra generated by $\hat{x}^\mu$ and devided by ideal generated by relations (1).

In other words, it consists of all polynomials of $\hat{x}^\mu$ and if one polynomial can be transformed to another one using (1) they are considered equal.

$$\begin{aligned} \hat{x}^1\hat{x}^2 &= [\hat{x}^1, \hat{x}^2] + \hat{x}^2\hat{x}^1 \\ &= \Theta^{12}(\hat{x}) + \hat{x}^2\hat{x}^1 \end{aligned}$$

Three forms of $\Theta^{\mu\nu}(\hat{x})$ of special interest

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}, \quad \theta^{\mu\nu} = -\theta^{\nu\mu} = const., \tag{2}$$

$$[\hat{x}^\mu, \hat{x}^\nu] = iC^{\mu\nu}_\lambda \hat{x}^\lambda, \quad C^{\mu\nu}_\lambda \text{ Lie algebra struct. const.,} \tag{3}$$

$$\hat{x}^\mu\hat{x}^\nu = \frac{1}{q} R^{\mu\nu}{}_{\rho\sigma}\hat{x}^\rho\hat{x}^\sigma \quad R \text{ matrix of quantum group.} \tag{4}$$

Derivatives

Commutative space ("usual" derivatives)

$$[\partial_\rho\,, x^\mu] = \delta^\mu_\rho, \qquad [\partial_\rho, \partial_\sigma] = 0,$$

$$\partial_\rho\Big(f(x)g(x)\Big) = \Big(\partial_\rho f(x)\Big)g(x) + f(x)\Big(\partial_\rho g(x)\Big) \quad (5)$$

Leibniz rule is undeformed; in terms of comultiplication

$$\Delta(\partial_\rho) = \partial_\rho \otimes 1 + 1 \otimes \partial_\rho \quad (6)$$

Deformed space: map $\hat{\partial}$ such that

$$\hat{\partial}: \quad \hat{\mathcal{A}}_{\hat{x}} \rightarrow \hat{\mathcal{A}}_{\hat{x}} \quad (7)$$

defined on coordinates

$$\begin{aligned}[][\hat{\partial}_\rho, \hat{x}^\mu] &= \delta^\mu_\rho + \underbrace{\text{additional terms}} \quad (8)\\ &= \delta^\mu_\rho + \sum_j A^{\mu\rho_1\ldots\rho_j}_\rho(\hat{x})\hat{\partial}_{\rho_1}\ldots\hat{\partial}_{\rho_j}\end{aligned}$$

it has to be consistent with (1) relations

$$\hat{\partial}_\rho\Big([\hat{x}^\mu, \hat{x}^\nu] - i\Theta^{\mu\nu}(\hat{x})\Big) = F\Big([\hat{x}^\mu, \hat{x}^\nu] - i\Theta^{\mu\nu}(\hat{x})\Big)\hat{\partial}_\rho = 0, \quad (9)$$

conditions on $A^{\mu\rho_1\ldots\rho_j}_\rho(\hat{x})$ follow from (9) and from (8) one calculates deformed (nontivial) Leibniz rule

$$\hat{\partial}_\rho\big(\hat{f}\hat{g}\big) = (\hat{\partial}_\rho\hat{f})\hat{g} + \hat{f}(\hat{\partial}_\rho\hat{g}) + \text{ additional terms} \quad (10)$$

Example (and from now on only)
θ-**deformed space** (2)

Ansatz $[\hat{\partial}_\rho, \hat{x}^\mu] = \delta^\mu_\rho + f^\mu_\rho(\hat{\partial}, \theta)$, then

$$\begin{aligned}
\hat{\partial}_\rho \hat{x}^\mu \hat{x}^\nu &= ([\hat{\partial}_\rho, \hat{x}^\mu] + \hat{x}^\mu \hat{\partial}_\rho)\hat{x}^\nu \\
&= (\delta^\mu_\rho + f^\mu_\rho(\hat{\partial}, \theta))\hat{x}^\nu + \hat{x}^\mu([\hat{\partial}_\rho, \hat{x}^\nu] + \hat{x}^\nu \hat{\partial}_\rho) \\
&= (\delta^\mu_\rho + f^\mu_\rho(\hat{\partial}, \theta))\hat{x}^\nu + \hat{x}^\mu(\delta^\nu_\rho + f^\nu_\rho(\hat{\partial}, \theta) + \hat{x}^\nu \hat{\partial}_\rho), \\
\hat{\partial}_\rho \hat{x}^\nu \hat{x}^\mu &= \ldots \\
&= (\delta^\nu_\rho + f^\nu_\rho(\hat{\partial}, \theta))\hat{x}^\mu + \hat{x}^\nu(\delta^\mu_\rho + f^\mu_\rho(\hat{\partial}, \theta) + \hat{x}^\mu \hat{\partial}_\rho)
\end{aligned}$$

and

$$\hat{\partial}_\rho \theta^{\mu\nu} = \theta^{\mu\nu} \hat{\partial}_\rho.$$

$$\hat{\partial}_\rho\Big([\hat{x}^\mu, \hat{x}^\nu] - i\theta^{\mu\nu}\Big) = \Big([\hat{x}^\mu, \hat{x}^\nu] - i\theta^{\mu\nu}\Big)\hat{\partial}_\rho, \tag{11}$$

is fulfiled with $f^\mu_\rho(\hat{\partial}, \theta) = 0$, therefore

$$[\hat{\partial}_\rho, \hat{x}^\mu] = \delta^\mu_\rho. \tag{12}$$

Also,

$$[\hat{\partial}_\mu, \hat{\partial}_\nu] = 0 \tag{13}$$

is compatible with (12),

$$\hat{\partial}_\sigma\Big([\hat{\partial}_\rho, \hat{x}^\mu] - \delta^\mu_\rho\Big) = \Big([\hat{\partial}_\rho, \hat{x}^\mu] - \delta^\mu_\rho\Big)\hat{\partial}_\sigma.$$

Leibniz rule can be worked out using (12)

$$\hat{\partial}_\rho\big(\hat{f}\hat{g}\big) = (\hat{\partial}_\rho\hat{f})\hat{g} + \hat{f}(\hat{\partial}_\rho\hat{g}) \tag{14}$$

or in the form of comultiplication

$$\Delta\hat{\partial}_\rho = \hat{\partial}_\rho \otimes 1 + 1 \otimes \hat{\partial}_\rho. \tag{15}$$

Comultiplication is compatible with the algebra of derivatives (13):

$$[\Delta\hat{\partial}_\mu, \Delta\hat{\partial}_\nu] = 0. \tag{16}$$

Relations (13) and (15) define the bialgebra of derivatives on θ-deformed space.

$\star$-product approach

idea: express everything in terms of commutative variables, but keep track of deformation

motivation: connection with the "real" world, possible experimental checks?

how to do it: Poincaré-Birkhoff-Witt (PBW) theorem

construction: ordering in the $\hat{\mathcal{A}}_{\hat{x}}$ has to be specified, for example, basis for symmetric ordering

$$\begin{aligned} :\hat{x}^\mu: &= \hat{x}^\mu, \\ :\hat{x}^\mu\hat{x}^\nu: &= \frac{1}{2}(\hat{x}^\mu\hat{x}^\nu + \hat{x}^\nu\hat{x}^\mu), \quad \text{etc.} \end{aligned} \tag{17}$$

from PBW theorem:

$$\begin{aligned} :\hat{x}^\mu: &\mapsto x^\mu, \\ :\hat{x}^\mu\hat{x}^\nu: &\mapsto x^\mu x^\nu, \quad \text{etc.} \end{aligned}$$

$$\begin{aligned} \hat{f}(\hat{x}) &= C_0 + C_{1\mu} :\hat{x}^\mu: + C_{2\mu\nu} :\hat{x}^\mu\hat{x}^\nu: + \dots \\ &\downarrow \\ f(x) &= C_0 + C_{1\mu}x^\mu + C_{2\mu\nu}x^\mu x^\nu + \dots \end{aligned} \tag{18}$$

Element

$$\begin{aligned}\hat{x}^\mu\hat{x}^\nu &= \frac{1}{2}(\hat{x}^\mu\hat{x}^\nu+\hat{x}^\nu\hat{x}^\mu)+\frac{1}{2}(\hat{x}^\mu\hat{x}^\nu-\hat{x}^\nu\hat{x}^\mu)\\ &= :\hat{x}^\mu\hat{x}^\nu: +\frac{i}{2}\theta^{\mu\nu}\end{aligned}$$

is mapped to space of commutative coordinates as

$$\hat{x}^\mu\hat{x}^\nu \mapsto x^\mu x^\nu+\frac{i}{2}\theta^{\mu\nu} \stackrel{\text{def}}{=} x^\mu\star x^\nu$$

$$\hat{x}^\nu\hat{x}^\mu \mapsto x^\mu x^\nu-\frac{i}{2}\theta^{\mu\nu} \stackrel{\text{def}}{=} x^\nu\star x^\mu$$

$$[\hat{x}^\mu,\hat{x}^\nu]\mapsto[x^\mu \stackrel{\star}{,} x^\nu]=i\theta^{\mu\nu}$$

Map can be extended to funcions as well

$$\hat{f}(\hat{x})\hat{g}(\hat{x})=\hat{f}\cdot\hat{g}(\hat{x})\mapsto f\star g(x), \tag{19}$$

where

$$f\star g\,(x)=e^{\frac{i}{2}\frac{\partial}{\partial x^\rho}\theta^{\rho\sigma}\frac{\partial}{\partial y^\sigma}}f(x)g(y)\Big|_{y\to x}, \tag{20}$$

or

$$f\star g\,(x)=\sum_{n=1}^{\infty}\left(\frac{i}{2}\right)^n\frac{1}{n!}\theta^{\rho_1\sigma_1}\ldots\theta^{\rho_n\sigma_n}\Big(\partial_{\rho_1}\ldots\partial_{\rho_n}f(x)\Big)\Big(\partial_{\sigma_1}\ldots\partial_{\sigma_n}g(x)\Big)$$

is Moyal-Weyl $\star$-product.

$\star$-product representation of derivatives

$$\begin{aligned} \hat{f} \cdot \hat{g}(\hat{x}) &\mapsto f \star g(x) \\ \hat{\partial}_\mu \hat{f}(\hat{x}) &\mapsto \underbrace{\partial_\mu \star}_{?} f(x) \end{aligned}$$

how to do it:

$$\begin{array}{ccc} \hat{f}(\hat{x}) & \longmapsto & f(x) \\ \hat{\partial}\downarrow & & \downarrow \partial\,\star \\ (\hat{\partial}\hat{f})(\hat{x}) & \longmapsto & (\partial \star f)(x) \end{array} \tag{21}$$

for derivatives defined by (12)

$$(\partial_\rho \star f)(x) = \partial_\rho f(x) \tag{22}$$

and

$$\partial_\rho \star \Big(f \star g\,(x) \Big) = \Big(\partial_\rho \star f(x) \Big) \star g(x) + f(x) \star \Big(\partial_\rho \star g(x) \Big) \tag{23}$$

With (20), (22) and (23) one has enough ingredients to be able to start doing some physics.

Symmetries

Different ways of classifing symmetries in physics

1. space-time and internal
2. global and local
3. discreet and continuous
4. . . .

concept of gauge theory can be transfered to noncommutative setting

$$\delta_\alpha \psi^0(x) \ = \ i\alpha(x)\psi^0(x) \to \delta_\alpha\psi = i\Lambda_\alpha \star \psi(x)$$
$$\cdots$$

abelian and nonabelian groups, enveloping algebra approach, SW map, . . .

corrections to the Standard Model have been found

what happens with space-time symmetries?

Commutative bialgebra of diffeomorphisms

general coordinate transformations

$$x^\mu \to x'^\mu = x^\mu + \xi^\mu(x) \tag{24}$$

scalar field: $\phi(x) \to \phi'(x') = \phi(x)$,

$$\delta_\xi^{cl}\phi(x) \stackrel{\text{def}}{=} \phi'(x) - \phi(x) = -\xi^\mu\partial_\mu\phi(x) \tag{25}$$

algebra of transformations

$$[\delta_\xi^{cl}, \delta_\eta^{cl}] = \delta_\xi^{cl}\delta_\eta^{cl} - \delta_\eta^{cl}\delta_\xi^{cl} = \delta_{[\xi,\eta]}^{cl} \tag{26}$$

where $\delta_{[\xi,\eta]}^{cl}\phi(x) = \big(\xi^\lambda\partial_\lambda\eta^\mu - \eta^\lambda\partial_\lambda\xi^\mu\big)\partial_\mu\phi(x)$

$$\begin{aligned}\delta_\xi^{cl}\Big(\phi_1(x)\phi_2(x)\Big) &= \Big(\delta_\xi^{cl}\phi_1(x)\Big)\phi_2(x) + \phi_1(x)\Big(\delta_\xi^{cl}\phi_2(x)\Big)\\ &= -\xi^\mu\partial_\mu\Big(\phi_1(x)\phi_2(x)\Big)\end{aligned} \tag{27}$$

or as a comultiplication

$$\Delta\delta_\xi^{cl} = \delta_\xi^{cl}\otimes 1 + 1\otimes\delta_\xi^{cl} \tag{28}$$

One can check that

$$[\Delta\delta_\xi^{cl}, \Delta\delta_\eta^{cl}] = \Delta\delta_{[\xi,\eta]}^{cl}. \tag{29}$$

Relations (26) and (28) define classical (undeformed) bialgebra of diffeomorphisms.

Deformed bialgebra of diffeomorphisms

idea: deforme transformations (25) in such a way that can be easily rewritten in terms of elements form $\hat{\mathcal{A}}_{\hat{x}}$, that is, lifted to the space of noncommutative coordinates

1st step: vector field $X_\xi = \xi^\mu(x)\partial_\mu$

$$
\begin{aligned}
X_\xi \star \phi(x) &\stackrel{\text{def}}{=} \xi^\mu(x) \star \partial_\mu \phi(x) \\
&= \xi^\mu(\partial_\mu \phi) + \frac{i}{2}\theta^{\rho\sigma}(\partial_\rho \xi^\mu)(\partial_\sigma \partial_\mu \phi) + \dots \\
&= \sum_n \left(\frac{i}{2}\right)^n \frac{1}{n!}\theta^{\rho_1\sigma_1}\dots\theta^{\rho_n\sigma_n} \\
&\qquad \Big(\partial_{\rho_1}\dots\partial_{\rho_n}\xi^\mu(x)\Big)\partial_{\sigma_1}\dots\partial_{\sigma_n}\partial_\mu\phi(x) \\
&\stackrel{\text{def}}{=} \Xi_X \phi(x) \qquad (30)
\end{aligned}
$$

where Ξ_X is higher order differential operator

$$
\begin{aligned}
\Xi_X &= \sum_n \left(\frac{i}{2}\right)^n \frac{1}{n!}\theta^{\rho_1\sigma_1}\dots\theta^{\rho_n\sigma_n} \\
&\qquad \Big(\partial_{\rho_1}\dots\partial_{\rho_n}\xi^\mu(x)\Big)\partial_{\sigma_1}\dots\partial_{\sigma_n}\partial_\mu \qquad (31)
\end{aligned}
$$

This defines the map from vector fields to higher order differential operators

$$X_\xi \mapsto \Xi_X. \qquad (32)$$

2nd step: inversion of this map, that is, find X_ξ such that

$$\xi\phi(x) = \xi^\mu(x)\partial_\mu\phi(x) = X_\xi \star \phi(x) \tag{33}$$

$$X_\xi = \xi^\mu(x)\partial_\mu - \frac{i}{2}\theta^{\rho\sigma}(\partial_\rho\xi^\mu(x))\partial_\sigma\partial_\mu + \ldots$$

to all orders one finds

$$\begin{aligned} X_\xi \;=\; & \sum_n \left(\frac{-i}{2}\right)^n \frac{1}{n!}\theta^{\rho_1\sigma_1}\ldots\theta^{\rho_n\sigma_n} \\ & \qquad \Big(\partial_{\rho_1}\ldots\partial_{\rho_n}\xi^\mu(x)\Big)\partial_{\sigma_1}\ldots\partial_{\sigma_n}\partial_\mu \end{aligned} \tag{34}$$

$$\begin{aligned} X_\xi \star \phi \;&=\; \xi^\mu \star (\partial_\mu\phi) - \frac{i}{2}\theta^{\rho\sigma}(\partial_\rho\xi^\mu) \star (\partial_\sigma\partial_\mu\phi) + \ldots \\ \downarrow & \\ \hat{X}_\xi\hat{\phi} \;&=\; \hat{\xi}^\mu(\hat{\partial}_\mu\hat{\phi}) - \frac{i}{2}\theta^{\rho\sigma}(\hat{\partial}_\rho\hat{\xi}^\mu)(\hat{\partial}_\sigma\hat{\partial}_\mu\hat{\phi}) + \ldots \end{aligned}$$

3rd step: deformed transformation law of a scalar field

$$\delta_\xi\phi(x) = -\xi^\mu\partial_\mu\phi(x) = -X_\xi \star \phi(x) \tag{35}$$

algebra of transformations

$$\delta_\xi\delta_\eta - \delta_\eta\delta_\xi = \delta_{[\xi,\eta]} \tag{36}$$

is the same as (26), where is the deformation?

4th step: request

$$\delta_\xi(\phi_1 \star \phi_2) = -\xi^\mu \partial_\mu(\phi_1 \star \phi_2) \tag{37}$$

that is, $\phi_1 \star \phi_2$ is again scalar field

$$\begin{aligned} \text{RHS} &= -\xi^\mu\big((\partial_\mu\phi_1)\star\phi_2 + \phi_1\star(\partial_\mu\phi_2)\big) \\ &\neq \underbrace{\big(-\xi^\mu(\partial_\mu\phi_1)\big)}_{\delta_\xi\phi_1}\star\phi_2 + \phi_1\star\underbrace{\big(-\xi^\mu(\partial_\mu\phi_2)\big)}_{\delta_\xi\phi_2} \end{aligned}$$

but

$$\delta_\xi(\phi_1\star\phi_2) = (\delta_\xi\phi_1)\star\phi_2 + \phi_1\star(\delta_\xi\phi_2) + \text{ additional terms} \tag{38}$$

up to first order one finds

$$\begin{aligned} \delta_\xi(\phi_1\star\phi_2) &= (\delta_\xi\phi_1)\star\phi_2 + \phi_1\star(\delta_\xi\phi_2) \qquad (39) \\ &\quad -\frac{i}{2}\theta^{\rho\sigma}\Big(\big((\partial_\rho\delta_\xi)\phi_1\big)\partial_\sigma\phi_2 + (\partial_\rho\phi_1)\big((\partial_\sigma\delta_\xi)\phi_2\big)\Big) \end{aligned}$$

with $(\partial_\rho\delta_\xi)\phi_1 = -(\partial_\rho\xi^\mu)\partial_\mu\phi_1$, or as a comultiplication

$$\begin{aligned} \Delta\delta_\xi &= \delta_\xi\otimes 1 + 1\otimes\delta_\xi \\ &\quad -\frac{i}{2}\theta^{\rho\sigma}\Big((\partial_\rho\delta_\xi)\otimes\partial_\sigma + \partial_\rho\otimes(\partial_\sigma\delta_\xi)\Big) \qquad (40) \end{aligned}$$

one can check that $\Delta\delta_\xi$ is consistent with the algebra (36)

$$[\Delta\delta_\xi, \Delta\delta_\eta] = \Delta\delta_{[\xi,\eta]} \tag{41}$$

5th step: summary

there exists deformed bialgebra of diffeomorphisms

$$\begin{aligned}\delta_\xi\delta_\eta - \delta_\eta\delta_\xi &= \delta_{[\xi,\eta]}\\ \Delta\delta_\xi &= \delta_\xi \otimes 1 + 1 \otimes \delta_\xi\\ &\quad -\frac{i}{2}\theta^{\rho\sigma}\Big((\partial_\rho\delta_\xi)\otimes\partial_\sigma + \partial_\rho\otimes(\partial_\sigma\delta_\xi)\Big)\end{aligned}$$

algebra sector is undeformed, while the comultiplication becomes deformed

it is the deformation of classical bialgebra of diffeomorphisms

Fields

defined by their transformation laws under general coordinate transformations (24)

scalar field

$$\begin{aligned} \delta_\xi \phi(x) &= -X_\xi \star \phi(x) \\ &= -\xi^\mu \star (\partial_\mu \phi) + \frac{i}{2}\theta^{\rho\sigma}(\partial_\rho \xi^\mu) \star (\partial_\sigma \partial_\mu \phi) + \ldots \end{aligned} \tag{42}$$

covariant vector field

$$\begin{aligned} \delta_\xi V_\nu &= -X_\xi \star V_\nu - X_\nu{}^\mu \star V_\mu \\ &= -\xi^\mu \star (\partial_\mu V_\nu) + \frac{i}{2}\theta^{\rho\sigma}(\partial_\rho \xi^\mu) \star (\partial_\sigma \partial_\mu V_\nu) \\ &\quad -(\partial_\nu \xi^\mu) \star V_\mu - \frac{i}{2}\theta^{\rho\sigma}(\partial_\rho \partial_\nu \xi^\mu) \star (\partial_\sigma V_\mu) + \ldots \end{aligned} \tag{43}$$

tensor fields analogously. . .

$$\begin{aligned} \delta_\xi(\phi_1 \star \phi_2) &= -X_\xi \star (\phi_1 \star \phi_2) \\ \delta_\xi(V_\nu \star \phi) &= -X_\xi \star (V_\nu \star \phi_2) - X_\nu{}^\mu \star (V_\mu \star \phi) \\ &\ldots \end{aligned}$$

because of (40)

Deformed Lorentz transformations

$$\begin{aligned}\delta_\xi\phi(x) &= -X_\xi\star\phi(x)\\ &= -\xi^\mu\star(\partial_\mu\phi)+\frac{i}{2}\theta^{\rho\sigma}(\partial_\rho\xi^\mu)\star(\partial_\sigma\partial_\mu\phi)+\dots\end{aligned}$$

if $\xi^\mu = x^\lambda {\omega_\lambda}^\mu$ and $\omega_{\lambda\mu} = -\omega_{\mu\lambda} = const.$, then

$$\begin{aligned}\delta_\xi\phi(x) &= -(x^\lambda{\omega_\lambda}^\mu)\star(\partial_\mu\phi)+\frac{i}{2}\theta^{\rho\sigma}{\omega_\rho}^\mu\star(\partial_\sigma\partial_\mu\phi)\\ &\stackrel{\text{def}}{=} \delta_\omega\phi(x)\end{aligned} \tag{44}$$

rewritten in terms of the elements from $\hat{\mathcal{A}}_{\hat{x}}$

$$\hat{\delta}_\omega = -\hat{x}^\lambda{\omega_\lambda}^\mu\hat{\partial}_\mu+\frac{i}{2}\theta^{\rho\sigma}{\omega_\rho}^\mu\hat{\partial}_\mu\hat{\partial}_\sigma \tag{45}$$

consistent with (2) relations

$$\begin{aligned}\hat{\delta}_\omega\,\Big([\hat{x}^\mu,\hat{x}^\nu]-i\theta^{\mu\nu}\Big) &= \Big([\hat{x}^\mu,\hat{x}^\nu]-i\theta^{\mu\nu}\Big)\hat{\delta}_\omega\\ &+\Big([\hat{x}^\rho,\hat{x}^\mu]-i\theta^{\rho\mu}\Big){\omega_\rho}^\nu+\Big([\hat{x}^\nu,\hat{x}^\rho]-i\theta^{\nu\rho}\Big){\omega_\rho}^\mu\end{aligned} \tag{46}$$

transformations close (undeformed) algebra

$$[\hat{\delta}_\omega, \hat{\delta}'_\omega] = \hat{\delta}_{[\omega,\omega']} \tag{47}$$

comultiplication is deformed

$$\Delta\hat{\delta}_\omega = \hat{\delta}_\omega \otimes 1 + 1 \otimes \hat{\delta}_\omega + \frac{i}{2}\big(\theta^{\rho\sigma}\omega_\sigma{}^\mu - \theta^{\mu\sigma}\omega_\sigma{}^\rho\big)\hat{\partial}_\mu \otimes \hat{\partial}_\rho \tag{48}$$

result was also found by Chaichian et al., hep-th/0408069 and by Koch et al., hep-th/0409012

from (48) one sees that deformed Lorentz transformations do not close bialgebra themselves but derivatives have to be included as well

transformation law of a derivative

$$[\hat{\delta}_\omega, \hat{\partial}_\rho] = \omega_\rho{}^\mu \hat{\partial}_\mu \tag{49}$$

the θ-deformed Poincaré bialgebra

$$[\hat{\partial}_\mu, \hat{\partial}_\nu] = 0, \quad [\hat{\delta}_\omega, \hat{\partial}_\rho] = \omega_\rho{}^\mu \hat{\partial}_\mu,$$

$$[\hat{\delta}_\omega, \hat{\delta}'_\omega] = \hat{\delta}_{[\omega,\omega']},$$

$$\Delta\hat{\partial}_\rho = \hat{\partial}_\rho \otimes 1 + 1 \otimes \hat{\partial}_\rho,$$

$$\Delta\hat{\delta}_\omega = \hat{\delta}_\omega \otimes 1 + 1 \otimes \hat{\delta}_\omega + \frac{i}{2}\big(\theta^{\rho\sigma}\omega_\sigma{}^\mu - \theta^{\mu\sigma}\omega_\sigma{}^\rho\big)\hat{\partial}_\mu \otimes \hat{\partial}_\rho$$

Summary and outlook

- it is possible to construct deformed bialgebra of diffeomorphisms using the metod of inverting the $\star$-product; deformed diffeomorphisms are deformation of classical diffeomorphisms;
  algebra is undeformed, but comultiplication changes

- tensor calculus established

- using formalism developed one can define covariant derivative and work out Christoffel symbols and in that way proceed towards deformed (general relativity) gravity, work in progres